Problem sheet 1, 07-04-2025

Problems coming from Chris Bowman's book *Diagrammatic algebra* are referenced as the preliminary January version of the book available to the participants of the course by sending out an email to me: langlois@uni-bonn.de

0. (Drill) Write the (24) elements of $\mathfrak{S}_4$ in diagram form and compute a few example.

1. Symmetric group Write a proof that the diagrammatic presentation $\mathfrak{S}_n$ shown in class is isomorphic to the symmetric group $S_n = \langle s_1, \dots, s_{n-1} \mid s_i^2 = (s_i s_{i+1})^3 = (s_i s_j)^2 = 1, |i-j| > 1 \rangle$ for all n.

2. 14-15 puzzle The *15-puzzle* is a sliding puzzle made out of 15 numbered blocks in a 4×4 grid with one empty cell. The 14-15 problem asks to reach Configuration B from Configuration A (see Figure 1). Prove that it is impossible, and that furthermore exactly half of the total number of configurations can be attained from Configuration A, and the other half, from Configuration B. [Bow25, Thm 2.3.1].

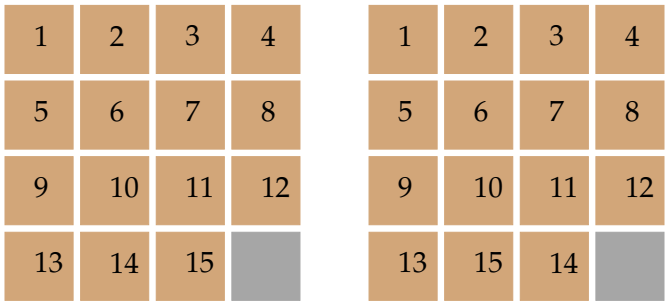

Figure 1: Two configurations of the 15-puzzle

3. [Bow25, Exercise 2.4.4, 2.4.5] Construct an isomorphisms $\phi : D_8 \to H_2$ for H_2 the hyper-octahedral group. Compute the order of H_n.

4. Coxeter graphs We can encode the Coxeter presentation in a graph; see [Bow25, Sect 2.6]. Give the Coxeter graph of the hyper-octahedral group H_n (So prove [Bow25, Proposition 2.6.3]).

5. Preparing the Temperley–Lieb algebra To prepare for the 14·04·2025 lecture, consider the following additional diagrammatic rules for $\mathfrak{S}_n^*$:

(1)

Compute the rule on all the 6 elements of $\mathfrak{S}_3$. What are your conclusions?

References

[Bow25] C. Bowman. *Diagrammatic algebra*. In press. Springer, 2025.