Diagrammatic algebra in representation theory

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One marker of representation theory in the 21st century is the strong influence of diverse fields, like categorical Lie theory, mathematical physics, homological algebra and knot theory. A thread common to those fields consists in viewing computation through some types of diagrammatic calculus. This point of view is now ubiquitous in representation theory and has informed many of the recent advances in the field.

Under the umbrella term *Diagrammatic algebra*, this course aims to popularise this diagrammatic insight through some concrete celebrated examples and through a diagrammatic version of Williamson's counterexamples to Lusztig's conjecture.

The main reference for this course will be Chris Bowman's book *Diagrammatic algebra* [Bow25], with further notions borrowed from Mathas [Mat99], Humphreys [Hum92] and Elias–Makisumi–Thiel–Williamson [Eli+20], along with examples from statistical mechanics models [Mar91].

Prerequisite Basic abstract algebra (group theory, rings, modules); basic representation theory, and some familiarity with categorical language and Lie theory.

Topics of the course

1. Preliminaries on algebras and their representations: reflection groups, composition series and the Jordan–Hölder theorem.
2. Non-semisimple representation theory: indecomposable and projective modules, idempotent truncation and submodule structure.
3. Quasi-hereditary and cellular algebras.
4. Temperley–Lieb algebras and their representation theory.
5. Diagrammatic algebras in physics: Affine Temperley–Lieb, Brauer, partition algebras, etc.
6. Basics of Kazhdan–Lusztig theory.
7. The Hecke categories and their diagrammatical bases (for parabolic Coxeter systems).
8. A sketch of Williamson's refutation of Lusztig's Conjecture.

References

[Bow25] C. Bowman. *Diagrammatic algebra*. In press. Springer, 2025.

[Eli+20] B. Elias, S. Makisumi, U. Thiel, and G. Williamson. *Introduction to Soergel bimodules*. Vol. 5. RSME Springer Ser. 2020. DOI: 10.1007/978-3-030-48826-0.

[Hum92] J. E. Humphreys. *Reflection groups and Coxeter groups*. 29. Cambridge university press, 1992.

[Mar91] P. Martin. *Potts models and related problems in statistical mechanics*. Vol. 5. Ser. Adv. Stat. Mech. Singapore etc.: World Scientific, 1991.

[Mat99] A. Mathas. *Iwahori-Hecke algebras and Schur algebras of the symmetric group*. Vol. 15. American Mathematical Soc., 1999.