Take up the lyre and quiver—you will be Apollo manifest

Apollonian poetry and quiver mutations

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Impressum

Preissue 1, 2026. Poetry and quiver mutation.

The title is Eleonora Cattafi's translation of Ovid's Sappho Heroid *Sume fidem et pharetram—fies manifestus Apollo* [Ovi71].

Contributors:

Eleonora Cattafi (Italian travel, p.20)

Álvaro Gutiérrez (El senado, p.12)

Alexis Langlois-Rémillard (Burning flies in Galway nights, p.9; Eyes open in concretle castle shadow p.14-15, D'un cycle à l'autre p.18)

Léo Schelstraete (D'un cycle à l'autre p.18)

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Associated with the presentation of Alexis Langlois-Rémillard at Bridges 2026, Galway, Ireland [Lan26].

We welcome contribution (in any language) to this issue until 2026/12/31.
`https://alexisl-r.github.io/popularization/mathpoetry/`

Alexis Langlois-Rémillard, Bonn, July 2026
`zine@tuta.com`

Poetry and quiver mutations

> *On me dira – et j'en conviens volontiers – qu'un poème n'est pas seulement l'addition des vers qui le composent. Quelque géométrie se mêle à cette arithmétique.*[1] (F. Le Lionnais, Préface à *Cent mille milliards de poèmes* de Raymond Queneau)

When writing a poem, rarely are we simply arranging individual lines to respect some arbitrary constraints. We embrace the form's constraints and write a whole text. The pages that follow present some examples of tentative rules and structures that try to inspire writers.

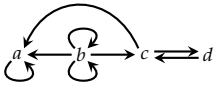
The rules are deeply mathematical, making use of quivers and their mutation; yet I also believe they are fun and accessible.

[1] It has been said, and I easily agree, that a poem is not only the addition of the lines that compose it. Some geometry intertwines this arithmetic.

Quivers and mutations

Quiver [Gab72] A *quiver* $Q = (V, E)$ is a directed graph with vertices in set V and directed edges, arrows, between vertices in set E. If an arrow has same source and target, it is a *loop*. If there are two arrows $a \to b$ and $b \to a$, we call them a *2-cycle*. Forgetting the orientation of a quiver gives its underlying graph, or *shape*.

This quiver has 4 vertices and 8 arrows. There are 3 loops (1 on vertex a and 2 on vertex b) and one 2-cycle between c and d



Quiver mutation [FZ03] Let $Q = (V, E)$ be a quiver without loops or 2-cycles. A *mutation* μ_v at the vertex $v \in V$ defines a new quiver $\mu_v(Q) = (V, \mu_v(E))$ on the same vertex set, with new arrows defined following the procedure below.

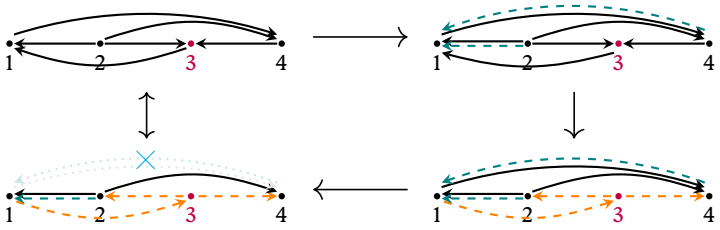
1. For every pair of arrows forming a path $u \to v \to w$ passing by v, draw a new arrow $u \to w$.

2. Reverse the orientation of each arrow $u \to v$ with target v and each arrow $v \to w$ with source v.

3. Remove the edges of all 2-cycles created by the previous.

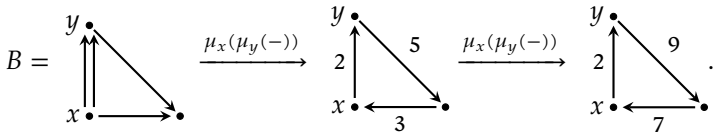
A quiver Q is mutation-equivalent to Q' if it can be obtained by successive applications of mutations $Q = \mu_{v_1}\mu_{v_2}\dots\mu_{v_k}(Q)$. The mutation class of Q is the collection of all quivers mutation-equivalent to Q.

For example below, the mutation at vertex "3" divided in three steps, modified arrows are in colours and dashed. Step 1 adds new arrows 2→1 and 4→1. Step 2 inverts three arrows with results: 3→2, 3→4 and 1→3. Step 3 removes the 2-cycle 1 ⇄ 4.



Mutating quivers can be a bit daunting. Luckily, mathematicians around the world have made available online applets to do it! Check out Bernhard Keller [Kel18] and Danielle Ensign [Ens25] applets.

Watchout! Some quivers can mutate indefinitely! For example



Apollonian quivers

The *Apollonian quiver* of a poem is the quiver constructed as follows.

1. The vertex set consists of the lines of the poem;
2. an arrow is drawn from a line to another if the source line contains a subject doing an action or being described in the target line.

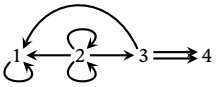
Where the fun starts

The second part of the process is where all the *art* is. Two people might disagree, especially for complex pieces. We can encode other information. Rhymes, for example, relation between or the number of syllables in verses.

La Pensée

Hiding in mazes of marble her chin sunk deep in the stone,
She breaks away from the senses five, the warders of the soul,
Alone in the wind-swept deeps of Being she seeks the Alone,
The adventurer's innermost light the dreamer's perilous goal.

Eva Gore-Booth [Gor04]

Apollonian quiver: 

1→1: stone→marble

2→1: She→hiding; 2→2: She→breaks

2→2: Senses→warders of the soul

2→3: She→seeks

3→1: deeps of Being→mazes of marble

$3 \xrightarrow{a} 4$: Alone$\xrightarrow{a}$adventurer's light

$3 \xrightarrow{b} 4$: Alone$\xrightarrow{b}$dreamer's goal

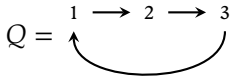
Apollonian processes

From a sole seed grew a forest Start from a stanza. Describe its Apollonian quiver. Mutate it on your favourite line, and write a new verse satisfying the new Apollonian quiver, keeping the line. Send it to another person if desired. Repeat until satisfied or if you go back to the initial Apollonian quiver.

Example Let's start with a haïku (a translation of one I wrote at Bridges' eve while looking at my rainy window).

1 August rains lit night
2 Flies swarm under the floodlight
3 Wings incadescent

Its Apollonian quiver is

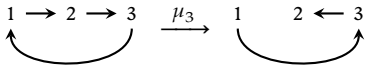


1 → 2 lit night—floodlight;

2 → 3 flies—wings;

3 → 1 Wings incadescent—lit.

This quiver Q has two mutation class shapes. So let me mutate it once to get the other shape.



Now let's write another haïku with the second quiver as Apollonian quiver. I am keeping the last line as I mutated on it.

Morning dew volupts
A moth crosses the window
Wings incadescent

Its Apollonian quiver is

$\mu_3(Q) =$

$1 \to 3$ volupts—incadescent;

$3 \to 2$ wings—moth;

Burning flies in Galway nights

August rains lit night
Flies swarm under the floodlight
Wings incadescent

Morning dew volupts
A moth crosses the window
Wings incadescent

Alexis Langlois-Rémillard, Galway 2026

Comment from the poet

The original (French) haïku came to me after looking at the rain through my window, mesmerized by the dance of light created by insect crossing a floodlight and by the almost smoke quality of the patterns. At first I thought they were just illuminated by the light. I then realised with the help of a friend that, for some of them, small Icarus, they were probably actually quite really on fire. The scene took another meaning, mutated. So I thought of mutating the last verse on which the figure of speech became reality.

Especially when writing collaboratively, I like to keep the mutated line in the next stanza, as it challenges you to keep improving individual lines (if the next person chooses the same line, it ends, since the mutation is involutive) while keeping a dense meaning.

It is not a fixed rule I'd say, and it can be dismissed at will.

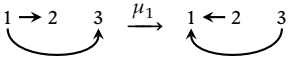
Second example Álvaro Gutiérrez proposed an example[2] starting by a verse with the following apollonian quiver

$$Q = \overset{1 \to 2 \quad 3}{\frown}.$$

1 ¡Oh! Vectores del bosque
2 que es magno y binario
3 que es alba y congreso

[2]He did not keep the mutated line, so it's a slightly freer form.

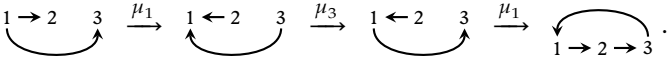
He then mutated the quiver on the first line to obtain a new quiver



and wrote the second stanza:

escuchen la proclama,
lince, liebre, marta,
rosal y jacaranda;

He repeated twice more and obtained the full poem. The complete mutation process is given below



The final poem, called *El senado*, the senate, is given next page.

Comment from the poet

The first stanza is a binary tree and it mentions a binary tree; the third mentions a path; the last is about a life cycle. The process is challenging, so I chose a very easy quiver. The quiver definitely drove the poem, the theme and everything.

El senado

¡Oh! Vectores del bosque
que es magno y binario
que es alba y congreso

escuchen la proclama,
lince, liebre, marta,
rosal y jacaranda;

que existe un sendero
que el viento nos traza
que es recto y que es justo:

por un canto la encina
amapolas de sombra cobija
que de colores al ave inspira.

Álvaro Gutiérrez May 2026

Process: Exhausted Apollo

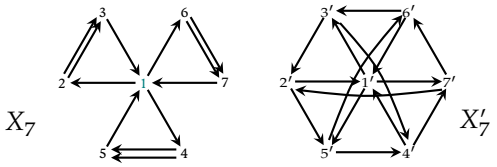
Take a mutation-finite quiver Q with n vertices. In particular, it has no loops or 2-cycles. For every shape appearing in the mutation class of Q, choose a representative quiver and write a verse of n lines whose Apollonian quiver matches it.

Mutation-finite quivers have been classified. The precise quivers are not really important, if you are interested, you can find them described in [DO08]. For a little history, the question of finding them was first asked by Fomin–Shapiro–Thurston [FST08, Problem 12.10], where they gave a list of potential candidates. Two more exceptional quivers were then found by Derken–Owen [DO08] and finally Felikson–Shapiro–Tumarkin [FST12] proved that the proposed list was complete.

Theorem ([FST12, Thm. 6.1] [DO08, Problem 15]). *A non-decomposable connected mutation-finite quiver either has 2 vertices, is of geometric type, or is mutation-equivalent to one of the eleven exceptional quivers* E_6, E_7, E_8, $\hat{E}_6$, $\hat{E}_7$, $\hat{E}_8$, $E_6^{(1,1)}$, $E_7^{(1,1)}$, $E_8^{(1,1)}$, X_6, X_7.

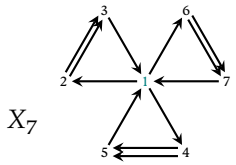
Example: the exceptional X_7 quiver

One of the extra exceptional quiver, X_7 is interesting: it has only two different shapes in its mutation classes. Here we choose X_7 and $X_7' = \mu_1\mu_3\mu_5\mu_7(X_7)$.



Eyes open in a concrete castle shadow

Marmoreal murder murmuring martyr
visions of naked poplars under stormy sky
frame cautious crows congregating
for the smell volutes and swirls
cold incense bathe the stone idols
listen, velocious droplets; voluptuous pebbles
harmony of an encloved cascade



X_7

1→2: martyr→vision; 1→4: martyr→smell; 1→6: martyr→listen

2→3: poplar→frame; 2→3: sky→frame

3→1: crows→congregating in a murder

4→5: volutes→incense 4→5: swirls→bath

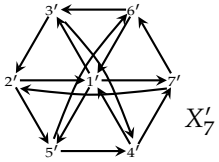
5→6: cold stone idols→marmoreal

6→7: droplets→cascade; 6→7: pebbles→cascade (goal)

7→1: cascade→murmuring

Murmuring martyr marmoreal murder
whispering waltzers wallowing whirlpool
hunts fox kindly, pelt of ruby soft shedding
clamours of statues dismantling socles past
races victim, moonlight floods the gates
crowds' hallali steals the breathes' bright
ascends in grace; the prey imposes its music

Alexis Langlois-Rémillard 2026



X_7'

$1' \to 3'$ martyr→hunts $1' \to 5'$: martyr→races $1' \to 7'$: martyr→ascends

$2' \to 1'$: waltzers→murmuring; $2' \to 5'$: whirlpooled→flood

$3' \to 2'$: fox→wallowing; $3' \to 4'$: hunts→clamours

$4' \to 1'$: statues→marmoreal; $4' \to 7'$: clamours→music

$5' \to 4'$: victim→dismantling; $5' \to 6'$ flood→steal breathes

$6' \to 1'$: hallali→murder; $6' \to 3'$: hallali→hunt

$7' \to 6'$: prey→hallali; $7' \to 2'$: music→waltzers

Analysis

The theme of the 14-line poem above is transformation, and it was written in two steps, before and after seeing an exhibition on Peter Hujar's photographic work in Bonn, *Eyes open in the dark*. The first verse incorporates an alchemical theme inspired by the symmetry of the quiver X_7: lines 2–3 go with air and with the vision; 4–5 with earth and touch; 6–7 with water and sound. The second verse expresses transformation and follows a dance of fire ($2'$–$5'$–$4'$–$7'$–$6'$–$3'$–$2'$) shortcuts by ether in $7'$–$2'$, $5'$–$6'$ and $3'$–$4'$ creating extra cycles. The middle verse transubstantiates meaning: the path $4'$–$1'$–$3'$ leaves statues shedding rubies, $2'$–$1'$–$7'$, waltzers turning musicians, and $6'$–$1'$–$5'$, the hallali shouts to the moon.

Dionysos

Upon reading the proposed forms, discontent sew in some readers and they wished for different liberties. Here is a different process created during discussion with Léo Schelstrate. It is, in some sense, *dual* to the Apollonian in that the lines of the poem are now represented in the arrows and the vertices are the concepts touched by the poem.

Dionysian seed Take a quiver without loops or 2-cycles. To each vertex, associate a concept. Arrows between two vertices represent a change from one concept to the other; the number of arrows is the number of lines in a poem that it takes to make the change. Write a stanza associated to the quiver. Then mutate the quiver and write another stanza from the mutated quiver. Continue the process until satisfied.

Dionysian path Consider the mutation class of a quiver. Choose two quivers in the mutation class. Find a series of mutations that comes from the first quiver to the second. Write a poem where each stanza has the corresponding Dionysian quiver from the mutation.

We give an example next page.

D’un cycle à l’autre

L’astre épris, tour sélène
Lune éternel frappe son glas
et se meurt en étreinte,

L’heure arrête au lit, douces,
nuits seules, les amours.

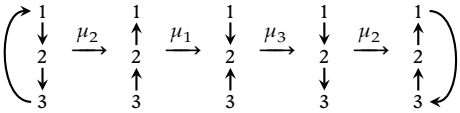
Les lèvres en baisers éphémères
leurs fins, un rien d’éternité

et leurs pétales fanent,
flane tout près la faux.

La rose tombe au creux
d’un sablier aux grains aimants ;
se voile étoile en veille.

Alexis Langlois-Rémillard et Léo Schelstraete 2026

Carquois



Procédé : Graine dionysiaque

Concepts : 1) Amour 2) Temps 3) Mort.

Forme : Haïku[3], structure initiale 6-8-6. Muté en trois distiques 6-6, 8-8, 6-6 et retour à un haïku 6-8-6.

Les flèches représentent les vers et indiquent une transformation du concept-source au concept-but.

Par exemple le premier vers « Astre épris, tour sélène » transfère le concept d'Amour vers celui du Temps dans l'image d'un astre amoureux en révolution (tour) lunaire. Le deuxième, du Temps vers la Mort, avec l'Éternité de la Lune et le glas qui annonce sa fin.

Un point intéressant de ce poème est qu'il contient plusieurs structure sous-poèmes dans les distiques : 4-6-8 forme un 6-8-6, mais aussi 4-6-9 et 4-7-8, etc.

[3]Nous avons un peu triché en prenant une structure 6-8-6 plutôt que le classique 5-7-5, un clin d'œil par une forme mutée et un peu plus libre que le haïku traditionnel

A sonnet of Italian travel

Dalla porta di casa al primo treno
mancano mezzi per tornare indietro
se della bici non partiva il freno
bloccati tutti gli autobus, la metro

dell'auto non avevo fatto il pieno
la moto aveva perso freno e vetro
sui tram che non partivano nemmeno
le porte si bloccavano da dietro

su tutti questi mezzi che perdiamo
solo il futuro parte e ci riporta
a casa come un fiume in piena al mare

e i mezzi che ci mancano scriviamo
in versi che spalancano la porta:
la casa la possiamo immaginare.

Eleonora Cattafi 2026

Comment of the author

Eleonora described it as a traditional (Italian) sonnet, verses are endecasyllables, structure of rhymes ABAB ABAB CDE CDE.
Recurring keywords: Porta (door): verses 1, 8, 13; Casa (home): 1, 11, 14 Mezzi (modes of transport): 2, 9, 12; Partire (leave): 3, 7, 10. Other keywords: mancare (miss, 2, 12), perdere (lose, 6,9), bloccare (block, 4,8), freno (brake, 3,6), pieno/a (full, 5,11), (in)dietro (back, 2,8).

Eleonora also provided the following "rough"[4] translation in English:

From the front door to the first train
there are no means of going back
if the bike's brake failed to engage,
all the buses and the metro were stalled

I hadn't filled the car's tank
the motorbike had lost its brake and glass
on the trams that wouldn't even leave
the doors were locked from the rear

of all these modes of transport we miss,
only the future departs and brings us back
home, like a river in flood to the sea

and the means of transport we lack, we write
in verses that throw the door wide open:
home is something we can imagine.

She mentionned she did not really follow my rules, but that she tried to enmesh the idea of mutation inside, from the concrete to the abstract. She was afraid the quiver of the poem would be quite sparse. When I tried to map the Apollonian quiver, I got something quite interesting in fact, with verses 9 and 12 being attractor and repulser respectively ("mezzi" referring to all the enumerated means of transport).

[4] According to her.

Hermès

One of the mathematical motivations of this work for me was to work a bit with the notation of *cluster algebra*. They are combinatorial objects defined by Fomin and Zelevinsky [FZ02]. One question I am exploring at the moment is how to include the notion into poetry writing. The first idea I had was to use the cluster basis to encode information on the metre of the poem. I have not been so successful, but maybe it is just to complicated for its purpose and we should stard with something more simple. Hence the following question.

Question Can you define a quiver associated with a stanza that also encodes the metre of the stanza? I would call such a quiver, a Hermès[5] quiver.

[5] In French, syllables in poem are called "pieds", feet, which explain the name hopefully.

Other contributions

These pages are yours to contribute. You can send your contributions (new forms, poems, drawing, etc.) to me with a name and the process you are following. I can give comments if you write in French, English, Dutch or German, but all languages are welcomed! Send me an email with object "Math poetry contribution" until 2026/12/31 at `zine@tuta.com`. The zine will be sent out by email to everybody who contributed (and maybe by snail mail to contributors if the postal service I find myself in are not too expensive).

`https://alexisl-r.github.io/popularization/mathpoetry/`

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Finally many thanks to the Bridges organisers for the conference, which was the impetus for this work.