

Take up the lyre and quiver—you will be Apollo manifest

Apollonian poetry and quiver mutations

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<https://alexisl-r.github.io/popularization/mathpoetry/>

Impressum

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The title is Eleonora Cattafi's translation of Ovid's Sappho Heroid *Sume fidem et pharetram—fies manifestus Apollo* [Ovi71].

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We welcome contribution (in any language) to this issue until 2026/12/31.

<https://alexisl-r.github.io/popularization/mathpoetry/>

Alexis Langlois-Rémillard, Bonn, July 2026
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Poetry and quiver mutations

*On me dira – et j'en conviens volontiers – qu'un poème n'est pas seulement l'addition des vers qui le composent. Quelque géométrie se mêle à cette arithmétique.*¹ (F. Le Lionnais, Préface à *Cent mille milliards de poèmes* de Raymond Queneau)

When writing a poem, rarely are we simply arranging individual lines to respect some arbitrary constraints. We embrace the form's constraints and write a whole text. The pages that follow present some examples of tentative rules and structures that try to inspire writers.

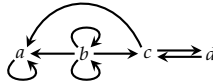
The rules are deeply mathematical, making use of quivers and their mutation; yet I also believe they are fun and accessible.

¹It has been said, and I easily agree, that a poem is not only the addition of the lines that compose it. Some geometry intertwines this arithmetic.

Quivers and mutations

Quiver [Gab72] A *quiver* $Q = (V, E)$ is a directed graph with vertices in set V and directed edges, arrows, between vertices in set E . If an arrow has same source and target, it is a *loop*. If there are two arrows $a \rightarrow b$ and $b \rightarrow a$, we call them a *2-cycle*. Forgetting the orientation of a quiver gives its underlying graph, or *shape*.

This quiver has 4 vertices and 8 arrows. There are 3 loops (1 on vertex a and 2 on vertex b) and one 2-cycle between c and d



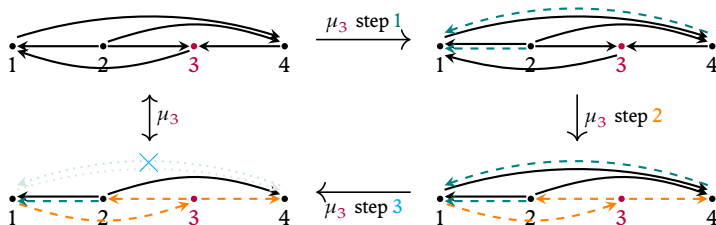
Quiver mutation [FZ03] Let $Q = (V, E)$ be a quiver without loops or 2-cycles. A *mutation* μ_v at the vertex $v \in V$ defines a new quiver $\mu_v(Q) = (V, \mu_v(E))$ on the same vertex set, with new arrows defined following the procedure below.

1. For every pair of arrows forming a path $u \rightarrow v \rightarrow w$ passing by v , draw a new arrow $u \rightarrow w$.

2. Reverse the orientation of each arrow $u \rightarrow v$ with target v and each arrow $v \rightarrow w$ with source v .
3. Remove the edges of all 2-cycles created by the previous.

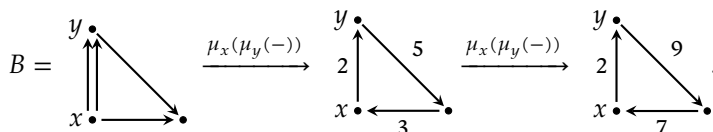
A quiver Q is mutation-equivalent to Q' if it can be obtained by successive applications of mutations $Q = \mu_{v_1} \mu_{v_2} \dots \mu_{v_k}(Q)$. The mutation class of Q is the collection of all quivers mutation-equivalent to Q .

For example below, the mutation at vertex “3” divided in three steps, modified arrows are in colours and dashed. Step 1 adds new arrows $2 \rightarrow 1$ and $4 \rightarrow 1$. Step 2 inverts three arrows with results: $3 \rightarrow 2$, $3 \rightarrow 4$ and $1 \rightarrow 3$. Step 3 removes the 2-cycle $1 \rightleftarrows 4$.



Mutating quivers can be a bit daunting. Luckily, mathematicians around the world have made available online applets to do it! Check out Bernhard Keller [Kel18] and Danielle Ensign [Ens25] applets.

Watchout! Some quivers can mutate indefinitely! For example



Apollonian quivers

The *Apollonian quiver* of a poem is the quiver constructed as follows.

1. The vertex set consists of the lines of the poem;
2. an arrow is drawn from a line to another if the source line contains a subject doing an action or being described in the target line.

Where the fun starts

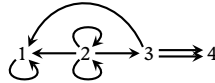
The second part of the process is where all the *art* is. Two people might disagree, especially for complex pieces. We can encode other information. Rhymes, for example, relation between or the number of syllables in verses.

La Pensée

- 1 Hiding in mazes of marble her chin sunk deep in the stone,
- 2 She breaks away from the senses five, the warders of the soul,
- 3 Alone in the wind-swept deeps of Being she seeks the Alone,
- 4 The adventurer's innermost light the dreamer's perilous goal.

Eva Gore-Booth [Gor04]

Apollonian quiver:

 $1 \rightarrow 1$: stone $\rightarrow$ marble $2 \rightarrow 1$: She $\rightarrow$ hiding; $2 \rightarrow 2$: She $\rightarrow$ breaks $2 \rightarrow 2$: Senses $\rightarrow$ warders of the soul $2 \rightarrow 3$: She $\rightarrow$ seeks $3 \rightarrow 1$: deeps of Being $\rightarrow$ mazes of marble $3 \xrightarrow{a} 4$: Alone $\xrightarrow{a}$ adventurer's light $3 \xrightarrow{b} 4$: Alone $\xrightarrow{b}$ dreamer's goal

Apollonian processes

From a sole seed grew a forest Start from a stanza. Describe its Apollonian quiver. Mutate it on your favourite line, and write a new verse satisfying the new Apollonian quiver, keeping the line. Send it to another person if desired. Repeat until satisfied or if you go back to the initial Apollonian quiver.

Álvaro Gutiérrez proposed an example² starting by a verse with the following apollonian quiver

$$Q = \begin{array}{ccc} 1 & \rightarrow & 2 & & 3 \\ & & \searrow & & \nearrow \end{array}.$$

*¡Oh! Vectores del bosque
que es magno y binario
que es alba y congreso*

He then mutated the quiver on the first line to obtain a new quiver

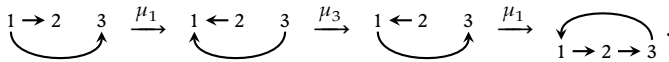
$$\begin{array}{ccc} 1 & \rightarrow & 2 & & 3 \\ & & \searrow & & \nearrow \end{array} \xrightarrow{\mu_1} \begin{array}{ccc} 1 & \leftarrow & 2 & & 3 \\ & & \nearrow & & \searrow \end{array}$$

²He did not keep the mutated line, so it's a slightly freer form.

and wrote the second stanza:

*escuchen la proclama,
lince, liebre, marta,
rosal y jacaranda;*

He repeated twice more and obtained the full poem. The complete mutation process is given below



Comment from the poet

The first stanza is a binary tree and it mentions a binary tree; the third mentions a path; the last is about a life cycle.

The process is challenging, so I chose a very easy quiver. The quiver definitely drove the poem, the theme and everything.

When writing collaboratively, I like to keep the mutated line in the next stanza, as it challenges you to keep improving individual lines (if the next person chooses the same line, it ends, since the mutation is involutive) while keeping a dense meaning.

El senado

1 ¡Oh! Vectores del bosque
2 que es magno y binario
3 que es alba y congreso

4 escuchen la proclama,
5 lince, liebre, marta,
6 rosal y jacaranda;

7 que existe un sendero
8 que el viento nos traza
9 que es recto y que es justo:

10 por un canto la encina
11 amapolas de sombra cobija
12 que de colores al ave inspira.

Álvaro Gutiérrez May 2026

Process: Exhausted Apollo

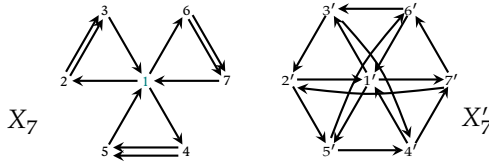
Take a mutation-finite quiver Q with n vertices. In particular, it has no loops or 2-cycles. For every shape appearing in the mutation class of Q , choose a representative quiver and write a verse of n lines whose Apollonian quiver matches it.

Mutation-finite quivers have been classified. The precise quivers are not really important, if you are interested, you can find them described in [DO08]. For a little history, the question of finding them was first asked by Fomin–Shapiro–Thurston [FST08, Problem 12.10], where they gave a list of potential candidates. Two more exceptional quivers were then found by Derksen–Owen [DO08] and finally Felikson–Shapiro–Tumarkin [FST12] proved that the proposed list was complete.

Theorem ([FST12, Thm. 6.1] [DO08, Problem 15]). *A non-decomposable connected mutation-finite quiver either has 2 vertices, is of geometric type, or is mutation-equivalent to one of the eleven exceptional quivers E_6 , E_7 , E_8 , $\hat{E}_6$, $\hat{E}_7$, $\hat{E}_8$, $E_6^{(1,1)}$, $E_7^{(1,1)}$, $E_8^{(1,1)}$, X_6 , X_7 .*

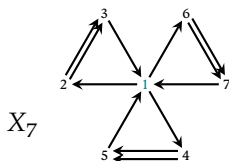
Example: the exceptional X_7 quiver

One of the extra exceptional quiver, X_7 is interesting: it has only two different shapes in its mutation classes. Here we choose X_7 and $X'_7 = \mu_1\mu_3\mu_5\mu_7(X_7)$.



Eyes open in a concrete castle shadow

- 1 Marmoreal murder murmuring martyr
- 2 visions of naked poplars under stormy sky
- 3 frame cautious crows congregating
- 4 for the smell volutes and swirls
- 5 cold incense bathe the stone idols
- 6 listen, velocious droplets; voluptuous pebbles
- 7 harmony of an enclosed cascade



1→2: martyr→vision; 1→4: martyr→smell; 1→6: martyr→listen

2→3: poplar→frame; 2→3: sky→frame

3→1: crows→congregating in a murder

4→5: volutes→incense 4→5: swirls→bath

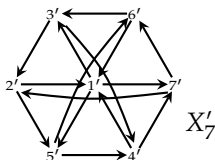
5→6: cold stone idols→marmoreal

6→7: droplets→cascade; 6→7: pebbles→cascade (goal)

7→1: cascade→murmuring

- 1 Murmuring martyr marmoreal murder
- 2 whispering waltzers wallowing whirlpool
- 3 hunts fox kindly, pelt of ruby soft shedding
- 4 clamours of statues dismantling socles past
- 5 races victim, moonlight floods the gates
- 6 crowds' hallali steals the breathes' bright
- 7 ascends in grace; the prey imposes its music

Alexis Langlois-Rémillard



1' → 3' martyr → hunts 1' → 5': martyr → races 1' → 7': martyr → ascends

2' → 1': waltzers → murmuring; 2' → 5': whirlpooled → flood

3' → 2': fox → wallowing; 3' → 4': hunts → clamours

4' → 1': statues → marmoreal; 4' → 7': clamours → music

5' → 4': victim → dismantling; 5' → 6' flood → steal breathes

6' → 1': hallali → murder; 6' → 3': hallali → hunt

7' → 6': prey → hallali; 7' → 2': music → waltzers

Analysis

The theme of the 14-line poem above is transformation, and it was written in two steps, before and after seeing an exhibition on Peter Hujar's photographic work in Bonn. The first verse incorporates an alchemical theme inspired by the symmetry of the quiver X_7 : lines 2–3 go with air and with the vision; 4–5 with earth and touch; 6–7 with water and sound. The second verse expresses transformation and follows a dance of fire ($2'-5'-4'-7'-6'-3'-2'$) shortcuts by ether in $7'-2'$, $5'-6'$ and $3'-4'$ creating extra cycles. The middle verse transubstantiates meaning: the path $4'-1'-3'$ leaves statues shedding rubies, $2'-1'-7'$, waltzers turning musicians, and $6'-1'-5'$, the hallali shouts to the moon.

Dionysos

Upon reading the proposed forms, discontent sew in some readers and they wished for different liberties. Here is a different process created during discussion with Léo Schelstrate. It is, in some sense, *dual* to the Apollonian in that the lines of the poem are now represented in the arrows and the vertices are the concepts touched by the poem.

Dionysian seed Take a quiver without loops or 2-cycles. To each vertex, associate a concept. Arrows between two vertices represent a change from one concept to the other; the number of arrows is the number of lines in a poem that it takes to make the change. Write a stanza associated to the quiver. Then mutate the quiver and write another stanza from the mutated quiver. Continue the process until satisfied.

Dionysian path Consider the mutation class of a quiver. Choose two quivers in the mutation class. Find a series of mutations that comes from the first quiver to the second. Write a poem where each stanza has the corresponding Dionysian quiver from the mutation.

We give an example next page.

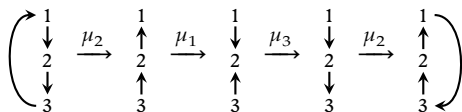
Exemples : Haïkus

D'un cycle à l'autre

- 1 L'astre épris, tour sélène
2 Lune éternel frappe son glas
3 et se meurt en étreinte,
- 4 L'heure arrête au lit, douces,
5 nuits seules, les amours.
- 6 Les lèvres en baisers éphémères
7 leurs fins, un rien d'éternité
- 8 et leurs pétales fanent,
9 flane tout près la faux.
- 10 La rose tombe au creux
11 d'un sablier aux grains aimants;
12 se voile étoile en veille.

Alexis Langlois-Rémillard et Léo Schelstraete 2026

Carquois



Procédé : Graine dionysiaque

Concepts : 1) Amour 2) Temps 3) Mort.

Forme : Haïku³, structure initiale 6-8-6. Muté en trois distiques 6-6, 8-8, 6-6 et retour à un haïku 6-8-6.

Les flèches représentent les vers et indiquent une transformation du concept-source au concept-but.

Par exemple le premier vers « Astre épris, tour sélène » transfère le concept d'Amour vers celui du Temps dans l'image d'un astre amoureux en révolution (tour) lunaire. Le deuxième, du Temps vers la Mort, avec l'Éternité de la Lune et le glas qui annonce sa fin.

Un point intéressant de ce poème est qu'il contient plusieurs structure sous-poèmes dans les distiques : 4-6-8 forme un 6-8-6, mais aussi 4-6-9 et 4-7-8, etc.

³Nous avons un peu triché en prenant une structure 6-8-6 plutôt que le classique 5-7-5, c'était représentait bien une forme muté et un peu plus libre que le haïku traditionnel

Hermès

One of the mathematical motivations of this work for me was to work a bit with the notation of *cluster algebra*. They are combinatorial objects defined by Fomin and Zelevinsky [FZ02]. One question I am exploring at the moment is how to include the notion into poetry writing. The first idea I had was to use the cluster basis to encode information on the metre of the poem. I have not been so successful, but maybe it is just too complicated for its purpose and we should start with something more simple. Hence the following question.

Question Can you define a quiver associated with a stanza that also encodes the metre of the stanza? I would call such a quiver, a Hermès⁴ quiver.

⁴In French, syllables in poem are called “pieds”, feet, which explain the name hopefully.

Other contributions

These pages are yours to contribute. You can send your contributions (new forms, poems, drawing, etc.) to me with a name and the process you are following. I can give comments in French, English, Dutch or German, but all languages are welcomed! Send me an email with object “Math poetry contribution” until 2026/12/31 at zine@tuta.com

<https://alexisl-r.github.io/popularization/mathpoetry/>



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