Apollonian poetry and quiver metamorphoses

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Abstract

We represent the structural properties of a given poem by using a quiver to describe relations between lines. We propose to use the process of quiver mutation to elaborate poetical creative processes and formal rules. They can be used collaboratively or in personal practice.

Introduction

Sume fidem et pharetram—fies manifestus Apollo[1]

From the elaborate metrics of medieval troubadours to the formal experimentations of the members of the *Ouvroir de Littérature Potentielle* (OuLiPo), mathematical structures have always been embedded in poetrical pursuits, as mystical mean, displays of virtuosity or challenges to peers. Following this tradition, we propose a new structure, which we find suitable for the creation, collaborative or personal, of dense meaningful pieces. It makes use of quivers and mutations thereof.

A quiver is, as its name hints, a collection of arrows. This tongue-in-cheek way of naming a directed graph was pioneered by Peter Gabriel in his work on the classification of finite-dimensional algebras [6]. Even if directed graphs were and still are heavily studied object, the interests of algebraists shed new light on them and produced new constructions to consider. The star of this note is quiver mutation, which was introduced in the early 2000's when Fomin and Zelevinsky initiated the study of cluster algebras [5].

In this short note, we define a quiver encoding relations between lines of a poem, which we name an Apollonian quiver, and we use quiver mutations to enter a playful dance with the text. We propose two poetical forms to showcase the creative potential of the constraints.

Quivers and their mutations

We define quiver and the vocabulary around it. An example of quivers is found in Figure 1.

Definition 1 (Quiver)**.** *A quiver Q consists of a set of vertices V and a set of arrows (or edges) $E = \{f : v_1 \to v_2 \mid v_1, v_2 \in V\}$. For an arrow $f : v_1 \to v_2$, we call v_1 the source of f and v_2, the target of f. We call rank, the cardinality of V. We call an arrow $f : v \to v$ boasting the same source and target a loop, and a pair of arrows $f : v_1 \to v_2$ and $f' : v_2 \to v_1$, a 2-cycle.*

Quivers have been around for quite a long time: one could even argue that they have been there since the inception of graph theory, because almost the first thing one wants to do on a graph is follow paths on it! Yet, a reason that makes them special in their own right is that they were defined to study finite-dimensional algebras, and, as is often the case, the interest from another field sparked the definition of somewhat arcane properties of the objects. Here we focus on quiver mutation, which was introduced by Fomin and Zelevinsky in their study of a class of polynomial algebras: cluster algebras [5].

[1]Take up the lyre and quiver—you will be Apollo manifest [9, Heroides 15.23]

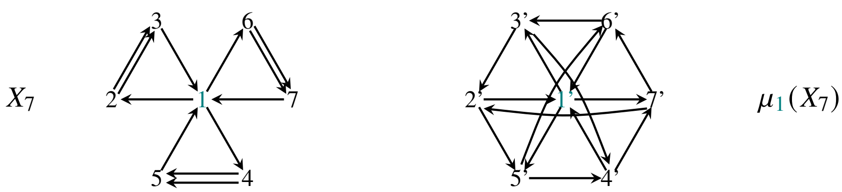


Figure 1: *Two quivers named X_7 and $\mu_1(X_7)$.*

Definition 2 (Quiver mutation)**.** *Let $Q = (V, E)$ be a quiver without loops and 2-cycles. A mutation μ_v at the vertex $v \in V$ is a new quiver $\mu_k(Q) = (V, \mu_k(E))$ on the same vertex set, with new arrows defined following the procedure below*

1. *for all pairs of arrows forming a path $u \to v \to w$ passing by v, draw a new arrow $u \to w$;*
2. *reverse the orientation of each arrow $u \to v$ with target v and each arrow $v \to w$ with source v;*
3. *remove all 2-cycles created by items 1 and 2.*

We call the set of all quivers created by successive applications of mutations at any point the mutation class of a quiver.

An example of quiver mutation is given in Figure 2. Also, the quivers of Figure 1 represent the two shapes of the mutation class of X_7: quivers obtained by mutations are made by choices of orientation on the two quivers X_7 and $\mu_1(X_7)$ (one might have to rearrange slightly the vertices). The readers are invited to mutate quivers by themselves; we suggest the following two browser applets to check [2, 8].

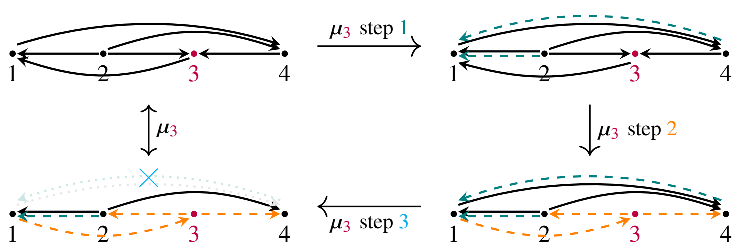
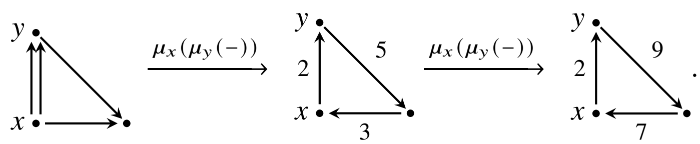


Figure 2: *Mutation at vertex "3" divided in three steps, modified arrows are in colours and dashed. Step 1 adds new arrows $2 \to 1$ and $4 \to 1$. Step 2 inverts three arrows with results: $3 \to 2$, $3 \to 4$ and $1 \to 3$. Step 3 removes the 2-cycle $1 \leftrightarrows 4$*

Some quivers are rather well behaved with respect to mutation, but it is not the case for all quivers. For example, in the quiver B (1), one can mutate successively vertices x and y to produce quivers with more and more arrows.

$B =$  . (1)

A natural question, first posed by Fomin–Shapiro–Thurston [4, Problem 12.10], is then to classify all the quivers with finite mutation classes. The full set was given in [1, Problem 15] and finally proven to be complete in [3]. We give X_7 in Figure 1, and we encourage the reader to seek the remainings in [1].

Theorem 3 ([3, Thm. 6.1] [1, Problem 15])**.** *A connected non-decomposable mutation-finite quiver either is of geometric type, has 2 vertices or is mutation-equivalent to one of the eleven exceptional quivers* $E_6, E_7, E_8, \widehat{E}_6, \widehat{E}_7, \widehat{E}_8, E_6^{(1,1)}, E_7^{(1,1)}, E_8^{(1,1)}, X_6, X_7$.

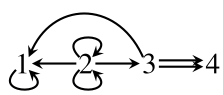
Apollonian mutation

We now need a way to encode some structure of a poem inside a quiver. There are many ways to do so, in fact, this is where art enters play: different choices will lead to different quivers suited for other purposes. After some tests, I found the following quiver to be the most fruitful to my practice. I call it the *Apollonian quiver* of a poem.

Definition 4. *The Apollonian quiver of a given poem is the quiver constructed as follows. 1) The vertex set is constituted by the lines of the poem; 2) an arrow is drawn from a line to another if the source line contains a subject doing an action or being described in the target line.*

We now look at a short poem by the 20th century Irish poet Eva Gore-Booth [7] that was inspired by Rodin's sculpture *La Pensée*. We can see its rich webs of meaning between the four lines.

Hiding in mazes of marble her chin sunk deep in the stone,
She breaks away from the senses five, the warders of the soul,
Alone in the wind-swept deeps of Being she seeks the Alone,
The adventurer's innermost light the dreamer's perilous goal.

Apollonian quiver: 

$1\to 1$: stone→marble
$2\to 1$: She→hiding; $2\to 2$: She→breaks
$2\to 2$: Senses→warders of the soul
$2\to 3$: She→seeks
$3\to 1$: deeps of Being→mazes of marble
$3\xrightarrow{a}4$: Alone$\xrightarrow{a}$adventurer's light
$3\xrightarrow{b}4$: Alone$\xrightarrow{b}$dreamer's soul

Apollonian metamorphoses

I propose to use the concept of mutation introduced in the previous section as a tool of playful interaction with poetry. I argue that it makes for a fruitful endeavour and encourages the creation of poems revealing in senses. As an example of how I used the concept, I propose two creative processes.

From a sole seed grew a forest Take a verse, either from a friend or from your favourite poet. Describe its Apollonian quiver. Mutate it on your favourite line, write a second verse satisfying the new Apollonian quiver and keeping the mutated line. Send it to another; repeat until satisfied or if you go back to the initial Apollonian quiver.

Commentary: I especially like this process because it gives you a challenge to keep improving individual lines (in order that the next person does not choose the same favourite line as you) while keeping a dense meaning. Also, mutation requires the quiver to have no loops and no 2-cycles. I see this as part of the genre constraints to foster more interdependent lines. A workaround can be to add extra vertices on culprit lines.

Exhausted Apollo Take a mutation-finite quiver of rank n, in particular, it has no loops and no 2-cycles. For every representative of the mutation classes, write a verse of n lines whose Apollonian quiver matches. Below is an example following X_7 and its other mutation class.

Commentary. The theme of the poem is transformation and was written in two steps, before and after seeing an art exhibition. The first verse incorporates an alchemical theme inspired by the symmetry of the quiver X_7: lines 2-3 go with air and with the vision; 4-5 with earth and touch; 6-7 with water and sound. The second verse expresses transformation and follows a dance of fire ($2'$-$5'$-$4'$-$7'$-$6'$-$3'$-$2'$) shortcuts by ether in $7'$-$2'$, 5′-6′ and 3′-4′ creating extra cycles. The middle verse transubstantiates meaning: the path 4′-1′-3′ leaves statues shedding ruby, 2′-1′-7′, waltzers turning musicians, and 6′-1′-5′, the hallali shout to the moon.

Marmoreal murder murmuring martyr $_1$
  visions of naked poplars under stormy sky $_2$
  frame cautious crows congregating $_3$
  for the smell volutes and swirls $_4$
  cold incense bathe the stone idols $_5$
  listen, velocious droplets; voluptuous pebbles $_6$
  harmony of an encloved cascade $_7$
Murmuring martyr marmoreal murder $_{1'}$
  whispering waltzers wallowing whirlpool $_{2'}$
  hunts fox kindly, pelt of ruby soft shedding $_{3'}$
  clamours of statues dismantling socles past $_{4'}$
  races victim, moonlight floods the gates $_{5'}$
  crowds' hallali steals the breathes' bright $_{6'}$
  ascends in grace; the prey imposes its music $_{7'}$

1→2: martyr→vision; 1→4: martyr→smell; 1→6: martyr→listen

2→3: poplar→frame; 2→3: sky→frame

3→1: crows→congregating in a murder

4→5: volutes→incense 4→5: swirls→bath

5→6: cold stone idols→marmoreal

6→7: droplets→cascade; 6→7: pebbles→cascade (goal)

7→1: cascade→murmuring

1′→3′ martyr→hunts1′→5′: martyr→races 1′→7′: martyr→ascends

2′→1′: waltzers→mumuring; 2′→5′: whirlpooled→flood

3′→2′: fox→wallowing; 3′→4′: →

4′→1′: statues→marmoreal; 4′→7′: clamours→music

5′→4′: victim→dismantling; 5′→6′ flood→steal breathes

6′→1′: hallali→murder; 6′→3′: hallali→hunt

7′→6′: prey→hallali; 7′→2′: music→waltzers

Conclusion and outlook

I described here two processes making used of the concept of quiver and mutation. I argue that the proposed poetical forms are filled with potential for dense meaning while also being playful in nature. I am currently experimenting to include the notion of cluster algebra in the format in order to add other constraints to the process. It can add weights to arrows and vertices in a combinatorially rich way.

Acknowledgements

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