Apollonian Poetry and Quiver Metamorphoses

Alexis Langlois-Rémillard

Hausdorff Center for Mathematics, Bonn, Germany; alexislr@tuta.com

Abstract

We represent the structural properties of a given poem by using a quiver to describe relations between lines. We propose to use the process of quiver mutation to elaborate poetical creative processes and formal rules. They can be used collaboratively or in personal practice.

Introduction

Sume fidem et pharetram—fies manifestus Apollo (Take up the lyre and quiver—you will be Apollo manifest) [9]

From the elaborate metrics of medieval troubadours to the formal experimentations of the members of the *Ouvroir de Littérature Potentielle* (OuLiPo), mathematical structures have always been embedded in poetical pursuits, as mystical means, displays of virtuosity, or challenges to peers. Following this tradition, we propose a new structure, which we find suitable for the creation, collaborative or personal, of dense, meaningful pieces. It makes use of quivers and mutations thereof.

A quiver is, as its name hints, a collection of arrows. This tongue-in-cheek way of naming a directed graph was pioneered by Gabriel in his work on the classification of finite-dimensional algebras [6]. Even if directed graphs were, and still are, heavily studied objects, the interests of algebraists shed new light on them and produced new constructions to consider. For example, quiver mutation was introduced in the early 2000's when Fomin and Zelevinsky initiated the study of cluster algebras [5].

In this paper, we define a quiver encoding relations between lines of a poem, which we name an *Apollonian quiver*, and we use quiver mutations to enter a playful dance with the text. We propose two poetical forms to showcase the creative potential of these quivers in poetry.

Quivers and their Mutations

Definition 1 (Quiver)**.** *A quiver $Q = (V, E)$ consists of a set of vertices V and a set of arrows (or edges) $E = \{f : v_1 \to v_2 \mid v_1, v_2 \in V\}$. For an arrow $f : v_1 \to v_2$, we call v_1 the source of f and v_2 the target of f. We call an arrow $f : v \to v$ boasting the same source and target a loop, and a pair of arrows $f : v_1 \to v_2$ and $f' : v_2 \to v_1$, a 2-cycle. Forgetting the orientation of a quiver gives its underlying graph, or shape.*

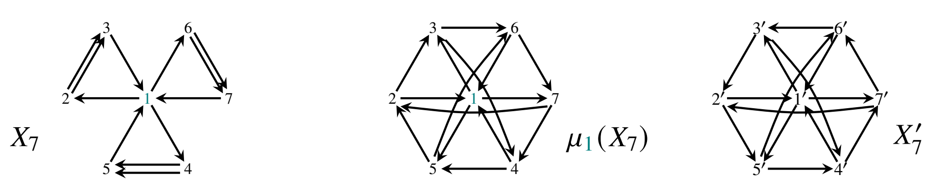


Figure 1: *Three quivers named X_7, $\mu_1(X_7)$ and X'_7. They are in the same mutation class, because $X'_7 = \mu_1\mu_3\mu_5\mu_7(X_7)$. Furthermore X'_7 and $\mu_1(X_7)$ have the same underlying graph.*

From the example in Figure 1, one might ask for the reason to redefine directed graphs. Gabriel argued for the new name because there were already too many notions associated with graphs. Quivers were defined to study finite-dimensional algebras, and, as is often the case, the new point of view sparked definitions of different properties than what graph theorists had studied until then. Here, we focus on quiver mutation.

Definition 2 (Quiver mutation)**.** *Let $Q = (V, E)$ be a quiver without loops or 2-cycles. A mutation μ_v at the vertex $v \in V$ defines a new quiver $\mu_v(Q) = (V, \mu_v(E))$ on the same vertex set, with new arrows defined following the procedure below.*

1. *For every pair of arrows forming a path $u \to v \to w$ passing by v, draw a new arrow $u \to w$.*
2. *Reverse the orientation of each arrow $u \to v$ with target v and each arrow $v \to w$ with source v.*
3. *Remove the edges of all 2-cycles created by items 1 and 2.*

A quiver Q' is mutation-equivalent to Q if it can be obtained by successive applications of mutations $Q' = \mu_{v_1}\mu_{v_2}\dots\mu_{v_k}(Q)$. The mutation class of Q is the collection of all quivers mutation-equivalent to Q.

Quiver mutation is involutive; applying it twice on the same vertex gives back the original quiver, that is, $\mu_v\mu_v(Q) = Q$. The order of application of mutations on different vertices, however, matters in general and can change the result. A step-by-step example of quiver mutation is given in Figure 2. Also, the three quivers of Figure 1 are all in the mutation class of X_7. The readers are invited to mutate quivers by themselves; we suggest the following two browser applets to play with the process [2, 8].

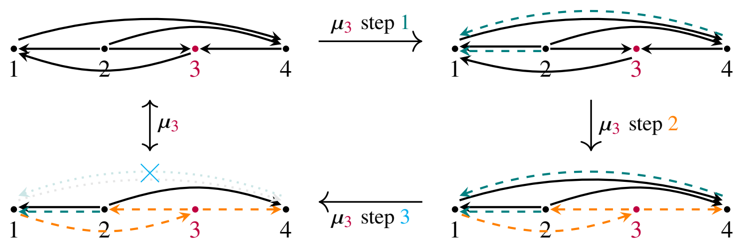
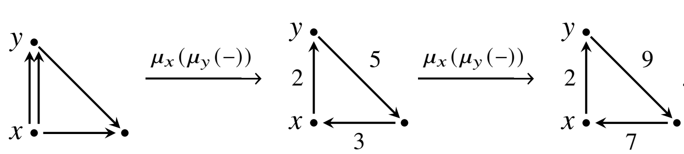


Figure 2: *Mutation at vertex "3" divided in three steps, modified arrows are in colours and dashed. Step 1 adds new arrows 2⟶1 and 4⟶1. Step 2 inverts three arrows with results: 3⟶2, 3⟶4 and 1⟶3. Step 3 removes the 2-cycle 1 ⇆ 4*

Some quivers are rather well behaved with respect to mutation, but it is not the case for all quivers. For example, in (1), the quiver B can be mutated successively at vertices x and y to produce quivers with more and more arrows, showing that its mutation class is infinite.

$B =$  (1)

A natural question, first posed by Fomin–Shapiro–Thurston [4, Problem 12.10], is to classify all quivers with finite mutation classes. If we can prove that there are finitely many underlying graphs in a class, then the class is finite, since the number of quivers of a given shape is finite. All quivers with finite mutation classes were first given in [1, Problem 15], adding two quivers X_6, X_7 to the initial list, and the upgraded list was proven to be complete in [3, Thm. 6.1]. We give X_7 and the only two shapes of its mutation class, X_7 and X_7', in Figure 1. A small warning that the underlying graphs do not remember the vertices' labels and so, when comparing two quivers, we can always reorganise the vertices to see if their shapes agree. For example $\mu_3\mu_1(X_7)$ has the same shape as X_7' by replacing $1' \mapsto 2, 2' \mapsto 4, 3' \mapsto 6, 4' \mapsto 1, 5' \mapsto 3, 6' \mapsto 7, 7' \mapsto 5$).

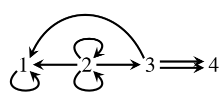
Apollonian Mutation

We now describe a way to encode some structure of a poem inside a quiver. There are many ways to do this, and it is where art comes into play. After some tests, I found the following quiver to be the most fruitful to my practice. I call it the *Apollonian quiver* of a poem.

Definition 3. *The Apollonian quiver of a given poem is the quiver constructed as follows. 1) The vertex set consists of the lines of the poem; 2) an arrow is drawn from a line to another if the source line contains a subject doing an action or being described in the target line.*

We now look at a short poem by the 20th century Irish poet Eva Gore-Booth [7] that was inspired by Rodin's sculpture *La Pensée*. We can see its rich webs of meaning between the four lines.

Hiding in mazes of marble her chin sunk deep in the stone, $_1$
She breaks away from the senses five, the warders of the soul, $_2$
Alone in the wind-swept deeps of Being she seeks the Alone, $_3$
The adventurer's innermost light the dreamer's perilous goal. $_4$

Apollonian quiver: 

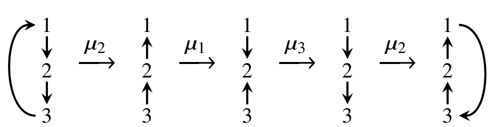
$1\to1$: stone$\to$marble
$2\to1$: She$\to$hiding; $2\to2$: She$\to$breaks
$2\to2$: Senses$\to$warders of the soul
$2\to3$: She$\to$seeks
$3\to1$: deeps of Being$\to$mazes of marble
$3\xrightarrow{a}4$: Alone$\xrightarrow{a}$adventurer's light
$3\xrightarrow{b}4$: Alone$\xrightarrow{b}$dreamer's goal

Apollonian Metamorphoses.

I propose to use the concept of mutation introduced in the previous section as a tool for playful interaction with poetry, illustrating it with two processes. I argue that it makes for a fruitful endeavour and encourages the creation of poems rich in meanings. Mutations require no loops or 2-cycles. I see this as part of the genre constraints to foster more interdependent lines, but they can be added by adding multiple vertices to a line.

From a sole seed grew a forest. Start from a verse. Describe its Apollonian quiver. Mutate it on your favourite line, write a verse satisfying the new Apollonian quiver, keeping the mutated line. Send it to another person; repeat until satisfied or if you go back to the initial Apollonian quiver.

Commentary. I like this process because it challenges you to keep improving individual lines (if the next person chooses the same line, it ends) while keeping a dense meaning. Below are the quivers coming from a haiku chain exchange I did with a colleague. The reader is encouraged to fill them with their own creations.



(2)

Exhausted Apollo. Take a mutation-finite quiver Q with n vertices. In particular, it has no loops or 2-cycles. For every shape appearing in the mutation class of Q, choose a representative quiver and write a verse of n lines whose Apollonian quiver matches it. The 14-line poem below is an example that I wrote taking $Q = X_7$ and using the two representatives X_7 and X_7' of the shapes of the mutation class of X_7 given in Figure 1.

Commentary. The theme of the poem is transformation and was written in two steps, before and after seeing an art exhibition. The first verse incorporates an alchemical theme inspired by the symmetry of the quiver X_7: lines 2–3 go with air and with the vision; 4–5 with earth and touch; 6–7 with water and sound. The second verse expresses transformation and follows a dance of fire ($2'$–$5'$–$4'$–$7'$–$6'$–$3'$–$2'$) shortcuts by ether in $7'$–$2'$,

$5'$–$6'$ and $3'$–$4'$ creating extra cycles. The middle verse transubstantiates meaning: the path $4'$–$1'$–$3'$ leaves statues shedding rubies, $2'$–$1'$–$7'$, waltzers turning musicians, and $6'$–$1'$–$5'$, the hallali shouts to the moon.

Marmoreal murder murmuring martyr $_1$
visions of naked poplars under stormy sky $_2$
frame cautious crows congregating $_3$
for the smell volutes and swirls $_4$
cold incense bathe the stone idols $_5$
listen, velocious droplets; voluptuous pebbles $_6$
harmony of an encloved cascade $_7$
Murmuring martyr marmoreal murder $_{1'}$
whispering waltzers wallowing whirlpool $_{2'}$
hunts fox kindly, pelt of ruby soft shedding $_{3'}$
clamours of statues dismantling socles past $_{4'}$
races victim, moonlight floods the gates $_{5'}$
crowds' hallali steals the breathes' bright $_{6'}$
ascends in grace; the prey imposes its music $_{7'}$

$1\to 2$: martyr→vision; $1\to 4$: mar.→smell; $1\to 6$: mar.→listen
$2\to 3$: poplar→frame; $2\to 3$: sky→frame
$3\to 1$: crows→congregating in a murder
$4\to 5$: volutes→incense $4\to 5$: swirls→bath
$5\to 6$: cold stone idols→marmoreal
$6\to 7$: droplets→cascade; $6\to 7$: pebbles→cascade (goal)
$7\to 1$: cascade→murmuring
$1'\to 3'$ martyr→hunts $1'\to 5'$: mar.→races $1'\to 7'$: mar.→ascends
$2'\to 1'$: waltzers→murmuring; $2'\to 5'$: whirlpooled→flood
$3'\to 2'$: fox→wallowing; $3'\to 4'$: hunts → clamours
$4'\to 1'$: statues→marmoreal; $4'\to 7'$: clamours→music
$5'\to 4'$: victim→dismantling; $5'\to 6'$ flood→steal breathes
$6'\to 1'$: hallali→murder; $6'\to 3'$: hallali→hunt
$7'\to 6'$: prey→hallali; $7'\to 2'$: music→waltzers

Conclusion and Outlook

I described here a way to associate an Apollonian quiver to a poem and presented two creative processes making use of quiver mutation. I argue that the proposed poetical forms are filled with potential for dense meaning while also being playful in nature. I welcome suggestions to explore the ideas further and am currently experimenting to include cluster algebra structures in order to add combinatorial constraints.

Acknowledgements

I wish to thank Joe Baine, Scott Neville, Léo Schelstraete and Ivan Sugrue for comments on an earlier draft, Eleonora Cattafi for providing the introductory quotation and its translation, and Danielle Ensign and Bernhard Keller for making their quiver mutation programs available to the community. Many thanks to the reviewers for their many comments that improved the idea and presentation. Supported by FRQNT (326641).

References

[1] H. Derksen and T. Owen. "New graphs of finite mutation type." *Electron. J. Comb.*, vol. 15, no. 1.
[2] D. Ensign. "Quiver mutation applet." 2025. https://www.phylliida.dev/quiver.html.
[3] A. Felikson, M. Shapiro, and P. Tumarkin. "Skew-symmetric cluster algebras of finite mutation type." *JEMS*, vol. 14, no. 4, 2012, pp. 1135–1180.
[4] S. Fomin, M. Shapiro, and D. Thurston. "Cluster algebras and triangulated surfaces. I: Cluster complexes." *Acta Math.*, vol. 201, no. 1, 2008, pp. 83–146.
[5] S. Fomin and A. Zelevinsky. "Cluster algebras II: Finite type classification." *Invent. Math.*, vol. 154, no. 1, 2003, pp. 63–121.
[6] P. Gabriel. "Unzerlegbare darstellungen I." *manuscripta math.*, vol. 6, no. 1, 1972, pp. 71–103.
[7] E. Gore-Booth. "La Pensée." *The One and The Many*. Longman, Green, and co., 1904.
[8] B. Keller. "Quiver mutation." 2018. https://webusers.imj-prg.fr/~bernhard.keller/quivermutation/.
[9] Ovid. "Epistula XV (Sappho Phaon)." *Ovidi Nasonis Epistulae Heroidum*. H. Dörrie, Ed. 1971.